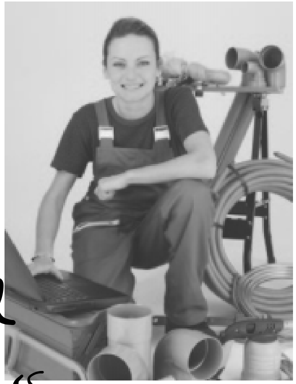
6.5 Comparing Properties of Linear Functions
page 293

Explore Comparing Properties of Linear Functions Given Algebra and a Description

Comparing linear relationships can involve comparing relationships that are expressed in different ways.



Dan's Plumbing and Kim's Plumbing have different ways of charging their customers. The function $D(t) = 35t$ represents the total amount in dollars that Dan's Plumbing charges for t hours of work. Kim's Plumbing charges $35 per hour plus a $40 flat-rate fee.

(A) Define a function $K(t)$ that represents the total amount Kim's Plumbing charges for t hours of work and then complete the tables.

K(t) = 35t + 40 represents the total amount that Kim's plumbing charges for t hours

Charges for Dan's Plumbing

t	$D(t) = 35t$	$(t, D(t))$
0 35(0)	0	(0,0)
1 35(1)	35	(1,35)
2 35(2)	70	(2,70)
3 35(3)	105	(3,105)

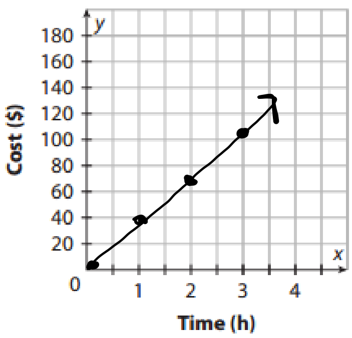
Charges for Kim's Plumbing

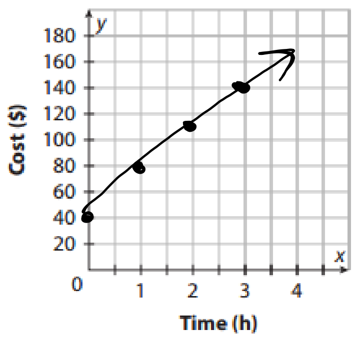
t	$K(t) =$	$(t, K(t))$
0	35(0)+40 = 40	(0,40)
1	35(1)+40 = 75	(1,75)
2	35(2)+40 = 110	(2,110)
3	35(3)+40 = 145	(3,145)

(B) What domain and range values for the functions $D(t)$ and for $K(t)$ are reasonable in this context? Explain.

Nonnegative numbers. A plumber can't work negative hours.

(C) Graph the two cost functions for the appropriate domain values.





(D) Compare the graphs. How are they alike? How are they different?

Same slope, different y-intercepts.

Reflect

1. **Discussion** What information could be found about the two functions without changing their representation?

the slope/rate of change.

Explain 1 Comparing Properties of Linear Functions Given Algebra and a Table

A table and a rule are two ways that a linear relationship may be expressed. Sometimes it may be helpful to convert one representation to the other when comparing two relationships. There are other times when comparisons are possible without converting either representation.

Example 1 Compare the initial value and the range for each of the linear functions $f(x)$ and $g(x)$.

(A) The domain of each function is the set of all real numbers x such that $5 \leq x \leq 8$. The table shows some ordered pairs for $f(x)$. The function $g(x)$ is defined by the rule $g(x) = 3x + 7$.

x	$f(x)$
5	20
6	24
7	28
8	32

The initial value is the output that is paired with the least input. The least input for $f(x)$ and $g(x)$ is 5.

The initial value of $f(x)$ is $f(5) = 20$.

The initial value of $g(x)$ is $g(5) = 3(5) + 7 = 22$.

Since $f(x)$ is a linear function and its domain is the set of all real numbers from 5 to 8, its range will be the set of all real numbers from $f(5)$ to $f(8)$. Since $f(5) = 20$ and $f(8) = 32$, the range of $f(x)$ is the set of all real numbers such that $20 \leq f(x) \leq 32$.

Since $g(x)$ is a linear function and its domain is the set of all real numbers from 5 to 8, its range will be the set of all real numbers from $g(5)$ to $g(8)$. Since $g(5) = 22$ and $g(8) = 3(8) + 7 = 31$, the range of $g(x)$ is the set of all real numbers such that $22 \leq g(x) \leq 31$.

(B) The domain of each function is the set of all real numbers x such that $6 \leq x \leq 10$. The table shows some ordered pairs for $f(x)$. The function $g(x)$ is defined by the rule $g(x) = 5x + 11$.

x	$f(x)$
6	36
7	42
8	48
9	54
10	60

The initial value is the output that is paired with the least input. The least input for $f(x)$ and $g(x)$ is 6.

The initial value of $f(x)$ is $f(6) = 36$

The initial value of $g(x)$ is $g(6) = 5(6) + 11 = 41$

Since $f(x)$ is a linear function, and its domain is the set of all real numbers from 6 to 10, its range will be the set of all real numbers from $f(6)$ to $f(10)$. Since $f(6) = 36$ and $f(10) = 60$ the range of $f(x)$ is the set of all real numbers such that $36 \leq f(x) \leq 60$

Since $g(x)$ is a linear function and its domain is the set of all real numbers from 6 to 10, its range will be the set of all real numbers from $g(6)$ to $g(10)$. Since $g(6) = 41$ and $g(10) = 5(10) + 11 = 61$, the range of $g(x)$ is the set of all real numbers such that $41 \leq g(x) \leq 61$

Reflect

2. **Discussion** How can you use a table of values to find the rate of change for a linear function?

Find difference of the y-values. Find the difference of the x-values. Divide change of y / change of x

Your Turn Slope

3. Find the rate of change for the linear function $f(x)$ that is shown in the table.

x	f(x)
3	22
4	29
5	36
6	43
7	50

+1 +1 +1 +1 ; +7 +7 +7 +7

$\frac{\text{change of } y}{\text{change of } x}$ $\frac{7}{1} = \boxed{7}$

4. The rule for $f(x)$ in Example 1B is $f(x) = 6x$. If the domains were extended to all real numbers, how would the slopes and y-intercepts of $f(x)$ and $g(x) = 5x + 11$ in Example 1B compare?

$f(x) = 6x$
$y = 6x + 0$
↑ Steeper

$g(x) = 5x + 11$
$y = 5x + 11$

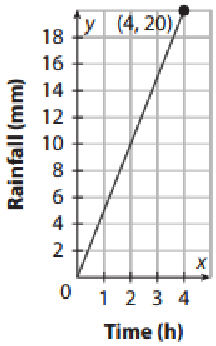
Explain 2 Comparing Properties of Linear Functions Given a Graph and a Description

Information about a linear relationship may have to be inferred from the context given in the problem.

Example 2 **Write a rule for each function, and then compare their domain, range, slope, and y-intercept.**

(A) A rainstorm in Austin lasted for 3.5 hours, during which time it rained a steady rate of 4.5 mm per hour. The function $A(t)$ represents the amount of rain that fell in t hours.

The graph shows the amount of rain that fell during the same rainstorm in Dallas, $D(t)$ (in millimeters), as a function of time t (in hours).



Write a rule for each function. $A(t) = 4.5t$ for $0 \leq t \leq 3.5$

The line representing $D(t)$ has endpoints at (0, 0) and (4, 20). The slope of $D(t)$ is $\frac{20-0}{4-0} = 5$. The y-intercept is 0, so substituting 5 for m and 0 for b in $y = mx + b$ produces the equation $y = 5x$. This can be represented by the function $D(t) = 5t$, for $0 \leq t \leq 4$.

The domains of each function both begin at 0 but end for different values of t, because the lengths of time that it rained in Austin and Dallas were not the same.

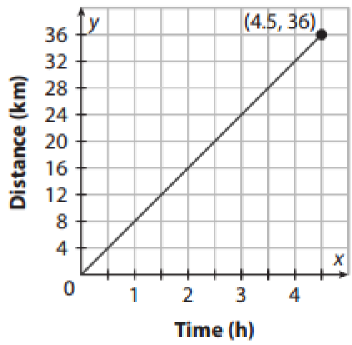
The range for $A(t)$ is $0 \leq A(t) \leq 15.75$. The range for $D(t)$ is $0 \leq A(t) \leq 20$.

The slope for $D(t)$ is 5, which is greater than the slope for $A(t)$, which is 4.5.

The y-intercepts of both functions are 0.

(B) One group of hikers hiked at a steady rate of 6.5 kilometers per hour for 4 hours. The function $f(t)$ represents the distance this group of hikers hiked in t hours.

The graph shows the distance a second group of hikers hiked, $g(t)$ (in kilometers), as a function of t (in hours).



Write a rule for each function.

$f(t) =$ 6.5t for 0 $\leq t \leq$ 4

The line representing $g(t)$ has endpoints at (0, 0) and (4.5, 36). The slope of $g(t)$ is $\frac{36 - 0}{4.5 - 0} =$ 8.

The y-intercept is 0, so substituting 8 for m and 0 for b in $y = mx + b$ produces the equation $y =$ 8x. This can be represented by the function $g(t) =$ 8t for 0 $\leq t \leq$ 4.5.

The domains of each function both begin at 0 and end at different values of t.

The range for $f(t)$ is 0 $\leq f(t) \leq$ 26 and the range for $g(t)$ is 0 $\leq g(t) \leq$ 36.

The slope for g(t) is greater than the slope for f(t)

The y-intercepts are both 0.

Reflect

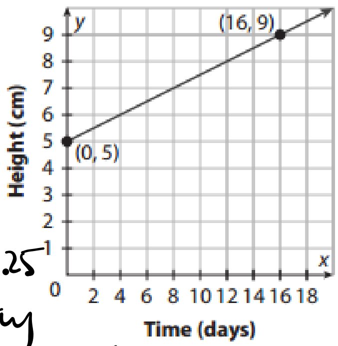
5. What is the meaning of the y-intercepts for the functions $A(t)$ and $D(t)$ in Example 2A?

The rainfall started at the same time that the rainstorm started. At that time, the total rainfall was 0 mm in both locations.

Your Turn

6. An experiment compares the heights of two plants over time. A plant was 5 cm tall at the beginning of the experiment and grew 0.3 centimeters each day. The function $f(t)$ represents the height of the plant (in centimeters) after t days. The graph shows the height of the second plant, $g(t)$ (in centimeters), as a function of time t (in days).

Find the rate of change $g(t)$ and compare it to the rate of change for $f(t)$.



Rate of Change $g(t)$: $\frac{9-5}{16-0} = 0.25$

The slope is 0.25 cm per day which is less than the slope of $f(t)$.

Elaborate

7. When would representing a linear function by a graph be more helpful than by a table?

8. When would representing a linear function by a table be more helpful than by a graph?

9. **Essential Question-Check-In** How can you compare a linear function represented in a table to one represented as a graph?

Homework:

pg 299-308 #

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Compare the initial value and the range for each of the linear functions $f(x)$ and $g(x)$.

1. The ~~domain of each function~~ is the set of all real numbers x such that $2 \le x \le 5$. The table shows some ordered pairs for $f(x)$. The function $g(x)$ is defined by the rule $g(x) = x + 6$.

x	f(x)
2	5
3	7
4	9
5	11

$g(x) = x + 6$

x	x+6	y
2	(2)+6	8
3	(3)+6	9
4	(4)+6	10
5	(5)+6	11

Domain: $2 \le x \le 5$

Range: $8 \le y \le 11$

f(x): D, R

Domain: $2 \le x \le 5$

Range: $5 \le y \le 11$

2. The domain of each function is the set of all real numbers x such that $8 \le x \le 12$. The table shows some ordered pairs for $f(x)$. The function $g(x)$ is defined by the rule $g(x) = 7x - 3$.

f(x): D, R

x	f(x)
8	34
9	38
10	42
11	46
12	50

$g(x) = 7x - 3$

x	7x−3	y
8	7(8)−3	53
9	7(9)−3	60
10	7(10)−3	67
11	7(11)−3	74
12	7(12)−3	81

D: $8 \le x \le 12$

R: $53 \le y \le 81$

Domain: $8 \le x \le 12$

Range: $34 \le y \le 50$

3. The domain of each function is the set of all real numbers x such that $-4 \le x \le -1$. The function $f(x)$ is defined by the rule $f(x) = 2x + 9$. The table shows some ordered pairs for $g(x)$.

x	g(x)
−4	10
−3	9
−2	8
−1	7

4. The domain of each function is the set of all real numbers x such that $0 \leq x \leq 4$. The function $f(x)$ is defined by the rule $f(x) = -3x + 15$. The table shows some ordered pairs for $g(x)$.

x	$g(x)$
0	23
1	19
2	15
3	11
4	7

5. The domain of each function is the set of all real numbers x such that $10 \leq x \leq 13$. The table shows some ordered pairs for $f(x)$. The function $g(x)$ is defined by the rule $g(x) = \frac{1}{2}x + 12$.

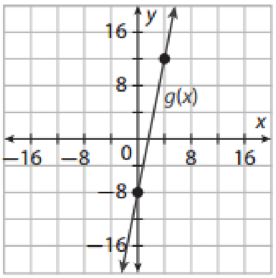
x	$f(x)$
10	22
11	$\frac{47}{2}$
12	25
13	$\frac{53}{2}$

6. The domain of each function is the set of all real numbers x such that $2 \leq x \leq 6$. The function $f(x)$ is defined by the rule $f(x) = -\frac{3}{4}x + 10$. The table shows some ordered pairs for $g(x)$.

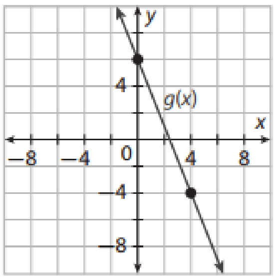
x	$g(x)$
2	14
3	$\frac{51}{4}$
4	$\frac{23}{2}$
5	$\frac{41}{4}$
6	9

Write a rule for each function f and g, and then compare their domains, ranges, slopes, and y-intercepts.

7. The function $f(x)$ has a slope of 6 and has a y-intercept of 20. The graph shows the function $g(x)$.



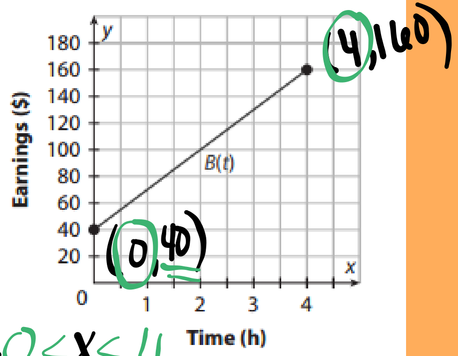
8. The function $f(x)$ has a slope of -3 and has a y-intercept of 5. The graph shows the function $g(x)$.



Write a rule for each function, and then compare their domains, ranges, slopes, and *y*-intercepts.

9. Jeff, an electrician, had a job that lasted 5.5 hours, during which time he earned \$32 per hour and charged a \$25 service fee. The function $J(t)$ represents the amount Jeff earns in t hours.

Brendan also works as an electrician. The graph of $B(t)$ shows the amount in dollars that Brendan earns as a function of time t in hours.

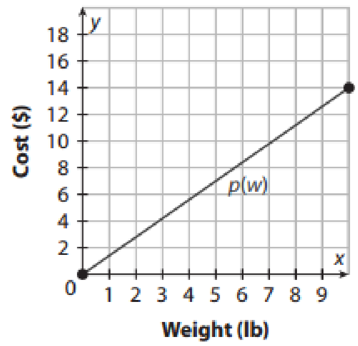


Domain: $0 \le x \le 4$

Range: $40 \le y \le 160$

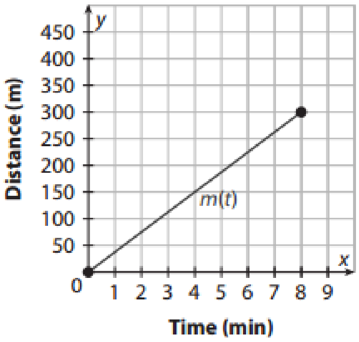
10. Apples can be bought at a farmer's market up to 10 pounds at a time, where each pound costs \$1.10. The function $a(w)$ represents the cost of buying w pounds of apples.

The graph of $p(w)$ shows the cost in dollars of buying w pounds of pears.



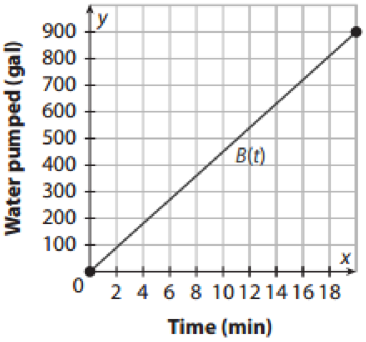
11. **Biology** A gecko travels for 6 minutes at a constant rate of 19 meters per minute. The function $g(t)$ represents the distance the gecko travels after t minutes.

The graph of $m(t)$ shows the distance in meters that a mouse travels after t minutes.



12. Cindy is buying a water pump. The box for Pump A claims that it can move 48 gallons per minute. The function $A(t)$ represents the amount of water (in gallons) Pump A can move after t minutes.

The graph of $B(t)$ shows the amount of water in gallons that Pump B can move after t minutes.



13. Erin is comparing two rental car companies for an upcoming trip. The function $A(d) = 0.20d$ represents the total amount in dollars of driving a car d miles from company A. Company B charges \$0.10 per mile and a \$10 fee.

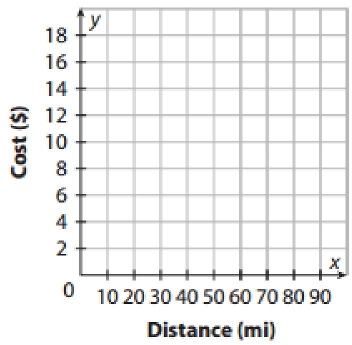
a. Define a function $B(d)$ that represents the total amount company B charges for driving d miles and then complete the tables.

Cost for Company A		
d	$A(d) = 0.20d$	$(d, A(d))$
0		
20		
40		
60		

Cost for Company B		
d	$B(d) =$	$(d, B(d))$
0		
20		
40		
60		

b. Graph and label the two cost functions for all appropriate domain values.

c. Compare the graphs. How are they alike? How are they different?



14. Snow is falling in two cities. The function $C(t) = 2t + 8$ represents the amount of snow on the ground, in centimeters, in Carlisle t hours after the snowstorm begins. There was 8 cm of snow on the ground in York when the storm began and the snow accumulates at 1.5 cm per hour.

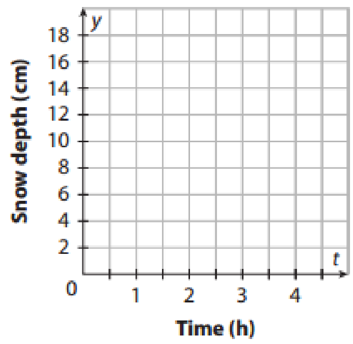
a. Define a function $Y(t)$ that represents the amount of snow on the ground after t hours in York and then complete the tables.

Carlisle		
t	$C(t) = 2t + 8$	$(t, C(t))$
0		
1		
2		
3		

York		
t	$Y(t) =$	$(t, Y(t))$
0		
1		
2		
3		

b. Graph and label the two cost functions for all appropriate domain values.

c. Compare the graphs. How are they alike? How are they different?



15. Gillian works from 20 to 30 hours per week during the summer. She earns $12.50 per hour. Her friend Emily also has a job. Her pay for t hours each given is given by the function $e(t) = 13t$, where $15 \leq t \leq 25$.

a. Find the domain and range of each function.

b. Compare their hourly wages and the amount they earn per week.

16. The function $A(p)$ defined by the rule $A(p) = 0.13p + 15$ represents the cost in dollars of producing a custom textbook that has p pages for college A, where $0 < p \leq 500$. The table shows some ordered pairs for $B(p)$, where $B(p)$ represents the cost in dollars of producing a custom textbook that has p pages for college B, where $0 < p \leq 500$. For both colleges, only full pages may be printed.

Compare the domain, range, slope, and y-intercept of the functions. Interpret the comparisons in context.

p	$B(p)$
0	24
50	30
100	36
150	42

17. Complete the table so that $f(x)$ is a linear function with a slope of 4 and a y-intercept of 7. Assume the domain includes all real numbers between the least and greatest values shown in the table. Compare $f(x)$ to $g(x) = 4x + 7$ if the range of $g(x)$ is $-1 \leq g(x) \leq 11$.

x	$f(x)$
−2	
−1	
0	
1	

18. Which functions have a rate of change that is greater than the one shown in the graph? Select all that apply.

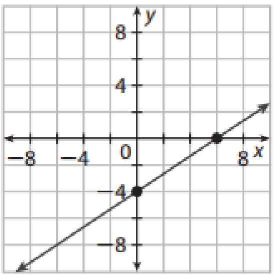
a. $f(x) = \frac{1}{2}x - 5$

b. $g(x) = -x + 6$

c. $h(x) = \frac{3}{4}x - 9$

d. $j(x) = -\frac{1}{4}x + 8$

e. $k(x) = x$



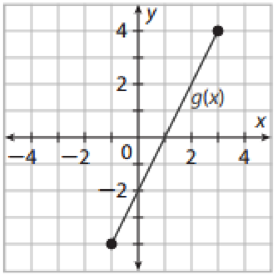
19. Does the function $f(x) = 5x + 5$ with the domain $6 \leq x \leq 8$ have the same domain as function $g(x)$, whose only function values are shown in the table? Explain.

x	$g(x)$
6	35
7	40
8	45

20. The linear function $f(x)$ is defined by the table, and the linear function $g(x)$ is shown in the graph. Assume that the domain of $f(x)$ includes all real numbers between the least and greatest values shown in the table.

a. Find the domain and range of each function, and compare them.

x	$f(x)$
−1	−7
0	−4
1	−1
2	2
3	5

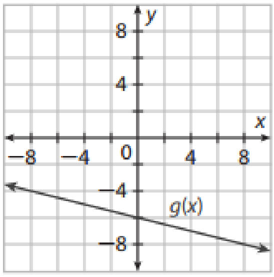


b. What is the slope of the line represented by each function? What is the y-intercept of each function?

21. The linear function $f(x)$ is defined by $f(x) = -\frac{1}{4}x + 6$ for all real numbers, and the linear function $g(x)$ is shown in the graph.

a. Find the domain and range of each function, and compare them.

b. What is the slope of the line represented by each function? What is the y-intercept of each function?



H.O.T. Focus on Higher Order Thinking

22. Communicate Mathematical Ideas Describe a linear function for which the least value in the range does not occur at the least value of the domain (a function for which the least value in the range is not the initial value.)

23. Draw Conclusions Two linear functions have the same slope, same x-intercept, and same y-intercept. Must these functions be identical? Explain your reasoning.

24. **Draw Conclusions** Let $f(x)$ be a line with slope -3 and y-intercept 0 with domain $\{0, 1, 2, 3\}$, and let $g(x) = \{(0, 0), (1, -1), (2, -4), (3, -9)\}$. Compare the two functions.

25. **Draw Conclusions** Let $f(x)$ be a line with slope 7 and y-intercept -17 with domain $0 \leq x \leq 5$, and let $g(x) = \{(0, -17), (1, -10), (2, -3), (3, 4), (4, 11), (5, 18)\}$. Compare the two functions.